PHOTOMETRIC MEASUREMENTS

OF THE

VARIABLE STARS

β PERSEI AND DM. 81°25,

MADE AT THE

HARVARD COLLEGE OBSERVATORY.

BY

EDWARD C. PICKERING, Director,
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INVESTIGATIONS ON LIGHT AND HEAT, PUBLISHED WITH AN APPROPRIATION FROM THE RUMFORD FUND.

XXI.

PHOTOMETRIC MEASUREMENTS OF THE VARIABLE STARS β PERSEI AND DM. 81°25, MADE AT THE HARVARD COLLEGE OBSERVATORY.

BY EDWARD C. PICKERING, DIRECTOR, ARTHUR SEARLE AND O. C. WENDELL, ASSISTANTS.

Presented April 13, 1881.

OUR knowledge of the cause of variation in the light of certain of the fixed stars must be derived largely from the curves showing the intensity of their light at any given time. Two methods may be employed for determining the form of these light-curves, as they are called. First, that proposed by Argelander, in which the variable is compared by the eye with some adjacent stars of nearly equal brightness. The difference, if any, is estimated in terms of a small unit called a grade, which nearly equals a tenth of a magnitude. A discussion of the entire series of measures serves to determine the light of the comparison stars, and to reduce all the measures to a scale of grades. This method is so simple, and gives results of such precision, that it has heretofore been almost exclusively used. For determining the form of the curve qualitatively, and the times of maximum and minimum light, this method leaves little to be desired. For a quantitative study of these curves, however, we must reduce the scale of grades to light ratios by photometric measures of the comparison stars. If, meanwhile, any of the comparison stars vary in light, errors are introduced which cannot be eliminated, and these, with the errors in the photometric measures, are likely to greatly exceed the errors in determining the form of the light-curve. The second method consists in a photometric measurement of the light of the variable at different times, and thus determining directly the form of its light-curve. Although the errors in the final results in the second method may be no larger than in the first, yet they are rendered much more conspicuous, so that hitherto no very satisfactory light-curves have been obtained in this way. On the

other hand, a photometric measure has a great advantage on the score of independence, as it may be repeated many times in succession in a single evening. An observer cannot repeat a naked-eye comparison many times without being prejudiced in the later measures by those which have preceded them.

The photometer used in the following measurements is essentially the same as that described in volume xi. of the Annals of the Harvard College Observatory, p. 4, figs. 1 and 2. A double-image prism is placed between the object-glass and eyepiece of a telescope, and in front of the eyepiece a Nicol prism is inserted. A circle divided into degrees serves to show the angle through which the Nicol is turned. Two adjacent stars may be compared by this instrument with great accuracy. Two images of each will be formed by the double-image prism, and their relative brightness varied at will by turning the Nicol. Each image in turn will disappear when the Nicol is revolved 90°. There will therefore always be four positions in which the brighter image of the fainter star will be precisely equal to the fainter image of the brighter star.

β Persei.

The measurements of *β Persei* were made by comparing it with *ω Persei*, a fifth magnitude star 90′ distant. The two images were formed by two Rochon prisms, which produced a separation of about 100′. It was therefore necessary that they should be placed very near the object-glass of the telescope, in order that the images of the two stars should be near together. The focal length of the telescope is about seventeen inches, and its available aperture is limited by the size of the prisms to about an inch. A larger aperture would be preferable, but is not very important on account of the brightness of the stars. The telescope is placed horizontally with a right-angled reflecting prism in front of the object-glass. The line of sight is therefore horizontal, even when objects in the zenith are observed, and the stars are followed by rotating the telescope around its axis, and also by turning the stand around a vertical axis. The four images are placed in line, and the two central ones, which are compared, are reversed by moving the prisms to and from the object-glass by a handle attached to the tube carrying them. This reversal was essential, and was always made after the second setting in each set. Errors due to the position of the images are thus completely eliminated. As the two images are seen on the same background under precisely the same conditions, many sources of error are eliminated. The conven-

ient position of the observer, the line of sight being always horizontal, also conduces to the accuracy of the measures. Owing to the low power used (about nine diameters), clockwork was not needed, the stars being occasionally brought back to the centre of the field as they moved away. The readings were wholly independent, as it was quite impossible to distinguish the images of the two stars when brought nearly to equality.

The four positions of the Nicol, in which the images appeared to be equal, were read by the graduated circle to tenths of a degree. This was repeated three times, so that twelve settings constituted a single set. Successive sets were commonly taken by different observers, but when one observer only was present he generally took care to leave the instrument for a minute or so between the sets, so that the same sources of error should not recur. Three observers took part in the work, Mr. Arthur Searle, Mr. O. C. Wendell, and myself. They will be designated by the initials S., W., and P.

Observations were made on thirteen nights, from Sept. 29, 1880, to Jan. 1, 1881, and will be detailed in a future volume of the Annals of the Observatory. The total number of sets was 230, containing 2,748 settings, two of the sets being incomplete. Twenty-eight sets were taken when the nearest minimum was five hours or more distant. They serve to determine the full brightness of the variable. Twelve sets by Mr. Searle give the excess in brightness of β over ω of 2.45 magnitudes; five by Mr. Wendell give 2.68; and twelve by myself give 2.67. As these results are confirmed by the other observations, we may conclude that ω appeared brighter or β fainter to Mr. Searle than to the other observers by about 0.22 magnitudes. All of his measures have been corrected by this amount to reduce all to the same system. Fifty sets, or six hundred readings, were obtained on Oct. 7, extending almost continuously from quarter of seven in the evening to half-past three of the following morning. On Oct. 10, nineteen sets were obtained from half-past six until nearly eleven, when clouds prevented further measures. On Nov. 2, forty-two sets were taken between six o'clock and midnight. On Nov. 19, fifteen sets were obtained; on Nov. 22, thirty-three; on Dec. 9, fourteen; on Dec. 22, nine; and on Jan. 1, twenty-eight. On Nov. 19, the observations of Mr. Searle appeared to differ from the results of the other observers by about three tenths of a magnitude, even after applying the correction of 0.22 magnitudes noted above, or without this correction they differed about half a magnitude. The reduction was first made retaining these, but they introduced so large a discord-

ance that the entire reduction has been repeated, rejecting them. No explanation can be offered for this difference, which occurs in nine sufficiently accordant sets. As the alternate observations of Mr. Wendell on the same evening agreed with the results of the other evenings, the effect seems to be due to the observer, and not to a variation of the star. The remaining observations have been arranged in groups according to the time preceding or following the minimum. Each group extends over half an hour, the computed minimum being the middle of one group. The first and last group extend from 255 to 345, the observations being more scattering. The results derived from these groups are given in Table I. The first column gives the mean of the times before or after the computed minimum. The latter was taken from the Annuaire of the Bureau des Longitudes, 1880, p. 78, which depends on the formula given by Schönfeld,* Ep. E = 1867 Jan. 0^d 11^h 1.2^m M Z Paris + 2^d 20^h 48.9^m (E — 8534).

The second column gives the number of sets in each group, and the third the mean of the observed magnitudes. The points defined by these times and magnitudes were then plotted on rectangular paper, and a smooth curve drawn nearly through them. Various precautions were taken to avoid small irregularities in this curve. The ordinates were read off, and the residuals computed from straight lines nearly tangent to the curve. These were plotted in turn, and the smooth curve drawn through them served to correct the original curve. The discussion of the rate of change in the light and of the true time of minimum, given below, also furnished small corrections, so that the curve should not only pass nearly through the observed points, but should undergo no sudden change in its direction or curvature. The ordinates of the final curve are given in the fourth column, and the deviations of the observations from them in the fifth column.

An inspection of this table shows that the observed minimum precedes that given by computation by more than half an hour. To determine the exact time of minimum, we must find the mean of the times when the light is equal. If the light curve was symmetrical, each of these means would equal the true minimum. Suppose that points are constructed with abscissas equal to the mean times, and ordinates to the corresponding light. Suppose that a smooth curve is drawn through these points, and extended to the point whose ordinate equals the light at the minimum. The abscissa of this point will give

* Sechsunddreissigster Jahresbericht des Mannheimer Vereins für Naturkunde, p. 94.

TABLE I.—LIGHT CURVE OF β PERSEI.

Time.	No.	Obs.	Curve.	O — C.
—	28	2.67	2.67	.00
— 278	8	2.68	2.67	+ .01
239	10	2.56	2.58	— .02
211	10	2.50	2.48	+ .02
181	9	2.29	2.31	— .02
151	11	2.15	2.14	+ .01
117	9	1.94	1.92	— .02
90	11	1.75	1.76	— .01
59	11	1.68	1.66	+ .02
— 29	11	1.64	1.64	.00
+ 1	17	1.70	1.70	.00
29	12	1.85	1.83	+ .02
63	9	1.98	2.00	— .02
91	12	2.19	2.17	+ .02
122	10	2.29	2.30	— .01
149	11	2.39	2.40	— .01
179	10	2.52	2.50	+ .02
212	10	2.59	2.58	+ .01
239	6	2.60	2.63	— .03
+ 275	6	2.66	2.66	.00
	221			± .013

the true minimum. Table II. gives for the various magnitudes contained in the first column the corresponding times at which the star attains this brightness during decrease and increase in the second and third column. The mean of these times is given in the fourth column.

TABLE II.—TIME OF MINIMUM.

Magn.	Dec.	Inc.	Mean.
2.6	— 246	+ 221	— 12.5
2.5	— 218	+ 180	— 19.0
2.4	— 196	+ 148	— 24.0
2.3	— 177	+ 122	— 27.5
2.2	— 161	+ 99	— 31.0
2.1	— 145	+ 80	— 32.5
2.0	— 131	+ 63	— 34.0
1.9	— 114	+ 45	— 34.5
1.8	— 97	+ 24	— 36.5
1.7	— 76	+ 1	— 37.5
1.68	— 70	— 5	— 37.5
1.66	— 60	— 13	— 36.5

From the above table we see that the true time of minimum preceded that given in the ephemeris by about thirty-seven minutes. An

ephemeris given by Dr. Schönfeld* for the present year differs by thirty-five minutes from his formula, or agrees within two minutes with the result of the present observations. The writer has shown in another place † that observations show a deviation from Schönfeld's formula of twenty-nine minutes at the end of 1878, and that this deviation is increasing at the rate of about three minutes a year, which would also give a correction of thirty-five minutes.

Any portion of the observations, as those of a single observer, or of one evening, would in general be better satisfied by moving the curve horizontally, or by assuming a different time of minimum. We wish, therefore, to know what correction t to the minimum is indicated by such observations. Let R equal the residual found by subtracting the value given by the assumed curve from that found by observation, and let r equal the residual when the minimum is altered by t. Also, let a equal the differential coefficient of the light in terms of the time, or the change of the light in magnitudes per minute. Then $R = r + at$, in which r and t are unknown. Solving with regard to t, we obtain, $t = \frac{R}{a} - \frac{r}{a}$. The weight to be assigned to such a determination of t will be proportional to a, since the errors are almost entirely due to erroneous determinations of the light, the error in the time being wholly insensible. Accordingly the effect on t of an error of a hundredth of a magnitude will be inversely as the rate of change of the light, or the weight should be proportional to a. Whatever the sign of a, the weight must always be positive. Multiplying the above value of t by a, we have $at = \pm R \pm r$, in which a is positive, and the signs of R and r will always be those of $\frac{R}{a}$ and $-\frac{r}{a}$. Taking the sum of all these equations, we obtain $\Sigma at = \Delta R - \Delta r$, in which Σ denotes the arithmetical sum of all the separate values, Δ their algebraic sum, taking into account the signs assigned them above. But $\Sigma at = t\Sigma a$, since, although t is unknown, it is the same for all the observations. Again, $\Delta r = 0$, since the separate values of r are arranged according to accident. Therefore, $t\Sigma a = \Delta R$, or $t = \frac{\Delta R}{\Sigma a}$. The computation is made by taking the algebraic sum of all values of R after changing the signs of those, in which a is negative, and dividing the result by the arithmetical sum of all the values of a.

* Vierteljahrsschrift, xv. 14. † Proceedings American Academy, xvi. 36.

The probable error, e, of the resultant value of t may be found from n, the number of residuals, their magnitudes r, and their weights a. The value of each expressed in minutes will be $\frac{r}{a}$, but since a weight of a should be assigned to it, we must write $a \times \frac{r}{a} = r$ The sum of all these terms will be Σr, and the sum of their weights Σa. The probable error will therefore be, $e = \frac{0.845\ \Sigma r}{\sqrt{n-1}\ \Sigma a}$. We cannot determine Σr directly, since r has not been computed. If t is not very large, ΣR will not greatly exceed Σr; we shall not therefore cause a large error if we write $e = \frac{0.845\ \Sigma R}{\sqrt{n-1}\ \Sigma a}$. The probable error thus found will be somewhat too large, so that the substitution from which it results cannot exaggerate the accuracy of the observations. $\frac{\Sigma R}{n}$ equals the average deviation D, and if n is large we may write $e = \frac{0.845\ \sqrt{n}\ D}{\Sigma a}$.

To apply this method we must determine the values of R and a corresponding to each set. The light corresponding to the time of each observation was read off from the light curve, and subtracted from the observed brightness. The value of a was determined as follows: A silk thread was kept stretched perfectly straight by making it the string to a bow of whalebone. It was then laid upon the curve so as to be tangent in turn to the points whose abscissas differ by twenty-five minutes. The ordinates of the points where the thread intersected two vertical lines, whose abscissas differed one hundred minutes, were next read. The difference in these ordinates, divided by one hundred, gave the change in magnitude per minute or a. Table III. gives, in the first and second columns, the corresponding times and values of a derived in this way, from the portion of the light curve preceding the minimum. Points were next plotted with the times as abscissas, and the values of a as ordinates, and a smooth curve drawn through them. The ordinates of this curve are given in the third column, and the residuals found from the observed values of a in the fourth column. The close agreement testifies to the smoothness of the curve and the precision of the measures. From the curve thus found, the values of a were read for each set. The last three columns correspond to the portion of the curve following the minimum.

TABLE III.—RATE OF CHANGE IN LIGHT.

Time.	Decreasing.			Increasing.		
	Obs.	Curve.	$O-C$.	Obs.	Curve.	$O-C$.
300	—	.0000	—	—	.0000	—
275	—	—.0014	—	—	+.0007	—
250	—.0030	—.0027	—.0008	—	+.0018	—
225	—.0037	—.0040	+.0008	+.0020	+.0019	+.0001
200	—.0054	—.0051	—.0008	+.0025	+.0026	—.0001
175	—.0058	—.0059	+.0001	+.0031	+.0031	.0000
150	—.0064	—.0063	—.0001	+.0035	+.0038	—.0003
125	—.0063	—.0063	.0000	+.0041	+.0045	—.0004
100	—.0056	—.0056	.0000	+.0048	+.0050	—.0002
75	—.0036	—.0036	.0000	+.0059	+.0054	+.0005
50	—.0011	—.0013	+.0002	+.0054	+.0054	.0000
25	+.0008	+.0010	—.0002	+.0046	+.0046	.0000
0	+.0036	+.0034	+.0002	—	—	—

This table also affords a method of determining the point of minimum light. At this point the rate of change should be zero, or should change from positive to negative. This evidently occurs between the times 50 and 25 minutes. Interpolating with the values given in either the second or third column gives for the exact time 36 minutes. This value agrees closely with 37 minutes, the value derived above from the points of equal light. The best method of determining the time of beginning and ending of the variation in light is from this same table. It will necessarily be subject to considerable error, since the observed curve must be extended according to the judgment of the observer. The times —300 and +300 have been found in this way. In other words, the star begins to diminish about 263 minutes before the minimum, and does not recover its original brilliancy until 337 minutes after. The most rapid diminution would occur at —140 or 100 minutes before the minimum. The variation would be then 0.0064 per minute.

The most rapid increase would occur at 100 minutes after the minimum, and would amount to 0.0055 magnitudes per minute.

In Table IV. the values of R and a are arranged in groups. A current number in the first column is followed in the second by the condition determining the groups. The next columns give the number of sets of twelve readings each, the arithmetical sum of the values of a, the arithmetical sum of the residuals, and their algebraic sum, giving to each the sign of R divided by a. The seventh column gives the correction to the assumed minimum found by dividing the sixth column by the fourth. The eighth column gives the probable error of the

resulting time, or $\frac{0.845\ \Sigma R}{\sqrt{n-1}\ \Sigma a}$. The last column but one gives the average residuals, or the fifth column divided by the third. The last column gives the average deviation of the three sets of four readings each, of which the sets of twelve readings are composed. It serves to show the accordance of the successive readings.

The first seven lines give the results for the seven minima which were observed. The next three lines group together all the observations of each observer. Lines 11 and 12 place together all those in which the light is decreasing, and those in which it is increasing. The results of all these sets is given in line 13. The rejected sets obtained on Nov. 19 are given in line 14. Line 15 groups those in which the star has its full brilliancy, or when the nearest minimum was more than five hours distant. The last line gives the results of lines 13 and 15, or the entire series, excepting those of Nov. 19. A set taken Oct. 7 is also included, which was taken so near the minimum that a was sensibly equal to zero. For this reason line 16 is not exactly equal to the sum of lines 13 and 15.

TABLE IV.—COMPARISON OF RESULTS.

No.	Group.	No. Sets.	Σa	ΣR	ΔR	$\frac{\Delta R}{\Sigma a}$	Prob. Err.	Av. Resid.	Av. Dev.
1	Oct. 7.....	49	.1937	4.00	−0.62	− 3.2	2.5	.081	.060
2	" 10.....	19	.0837	2.30	+0.28	+ 3.5	5.5	.121	.047
3	Nov. 2.....	43	.1284	2.64	−0.60	− 5.0	2.7	.061	.066
4	" 19.....	6	.0335	0.27	−0.11	− 3.6	3.0	.045	.056
5	" 22.....	33	.1362	3.32	+2.28	+17.5	2.6	.001	.083
6	Dec. 9.....	13	.0353	1.01	+0.11	+ 3.6	7.0	.078	.078
7	Jan. 1.....	28	.1214	2.47	−0.08	− 0.2	3.3	.088	.067
8	Obs. of P.	86	.3103	6.33	−0.35	− 1.2	1.9	.073	.063
9	" S.	45	.1864	5.30	+1.94	+10.4	2.9	.117	.085
10	" W.	60	.2355	4.38	−0.28	− 1.2	2.0	.073	.057
11	Decrease ..	81	.3342	7.27	+0.17	+ 0.5	2.1	.089	.066
12	Increase ..	110	.3980	8.74	+1.14	+ 2.9	1.6	.079	.066
13	Total......	191	.7322	16.01	+1.31	+ 1.8	1.3	.083	.066
14	Nov. 19. S.	9	.0385	3.55	−3.55	[− 92.2]	[27.8]	[.394]	.092
15	Full Light..	28	.0000	3.04	+0.06	—	—	.108	.068
16	Total......	220	.7322	19.14	+1.28	—	—	.086	.066

The observations of Nov. 22 show a large correction to the minimum. This is not easily explained unless the deviation is real. The measures before the minimum give a correction of + 15 minutes; those after, of + 18; those of P. alone, + 26; of S., + 14; of W., + 12. As the probable error of the mean result is only 2.6 minutes, and a nearly equal number of measures were made on each side of

the minimum, it is difficult to understand what instrumental errors could have caused the deviation. Including this minimum, the mean deviation for the seven nights is 5.2 minutes, or excluding the observations of Nov. 22, 3.2 minutes, the corresponding probable error would equal 4.7 and 3.0 minutes. The mean of the probable errors given in the next column is 3.8 minutes. This compares favorably with the results of naked-eye observations. Schmidt * gives the probable error of a single minimum observed by Argelander to be 6.0 minutes; of those of Schönfeld, 4.6 minutes; and of those by himself, 8.0 minutes. Probably still better results could have been obtained photometrically had the observations been designed for determining the time of minima. The mean of the whole series of measures would imply a correction to the adopted curve of + 1.8 minutes, with a probable error of 1.3 minutes. But if the observations of Nov. 22 are rejected, the correction becomes — 1.6 minutes. It therefore seems better to retain the correction to the ephemeris of 37 minutes, already adopted.

We have now a means of determining more precisely the constant difference between the different observers. The differences so far assumed are, P = 0.00, S = — 0.22, and W = 0.00 magnitudes. If either observer had taken an equal number of observations before and after the minimum, — or more strictly, if the weight of his observations before and after the minimum were equal, — an error in this correction would not affect the result. It would, however, very slightly exaggerate the residuals, and consequently the probable errors. If these personal differences were zero, the algebraic sum of the residuals of each observer should be zero. In fact, their values for the three observers are, for P = — 2.43, for S = + 1.66, and for W = + 1.84. As the total number of sets in the three cases are 98, 57, and 64, we obtain by division the deviations — 0.02, + 0.03, and + 0.03. Combining with these the correction of 0.22 already derived from Mr. Searle's observations of the full light of the variable, we find that the correction required to reduce his measures to mine is + 0.17, and to reduce Mr. Wendell's — 0.05, magnitudes. The effect of these changes on the final result would probably be wholly insensible.

Line 14 of the above table shows clearly that the observations of Nov. 19 should be rejected. They would indicate an error of an hour and a half in the minimum, if the deviations were not so large that the present method could not be applied.

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* Astron. Nach., lxxxvii. 204.

A comparison may now be made with the light-curve given by Schönfeld in the paper cited above. As has been already stated, the great difficulty lies in deciding what brightness shall be assumed for the comparison stars. In a previous article,* the light of these stars in grades assumed by Schönfeld have been reduced by means of the formula $L = 8.446 + 0.025\,g$, in which L gives the light and g the number of grades. This formula is derived from a comparison with the measurement of the comparison stars by Seidel and Wolff. These stars have since been measured with the meridian photometer of the Harvard College Observatory. Each star has been observed on the average on ten nights.

Three methods of reducing the grades of Schönfeld by these stars may be used. We may adopt the formula given above, which was found by a least square solution of the measures of Seidel and Wolff. Secondly, we may apply the method of least squares to the Harvard College Observatory measures, and deduce the formula most nearly satisfying them. This gives the value of one grade in logarithms equal to 0.029. In both these cases we have assumed that the value of a grade is the same for bright and for faint stars, and that the deviations are due to accidental errors, or to variations which have taken place in the light of the stars. As a third method, we may draw a curve through the points whose co-ordinates equal the light in grades and the measured brightness, and reduce the grades by means of this curve. We now assume that the errors are unimportant, but that the grade varies in different parts of the scale.

Table V. gives, in successive columns, the name of the star, its light in grades, the number of nights on which it has been observed at Cambridge, the resulting magnitude, the probable error, and the logarithm of the light, adopting the same unit as that of Seidel. Observations of *β Persei* have been included in this list, excluding those made within a few hours of its minimum. Three columns of residuals exhibit differences between the measures of Seidel, of Wolff, and of the Harvard College Observatory, and the values computed by the formula $L = 8.446 + 0.025\,g$. The next column gives the H. C. measures reduced to logarithms, minus those given by the formula $8.391 + 0.029\,g$. The last column gives the difference between the measures of the stars and the values of their light derived from the smooth curve.

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* Proc. Am. Acad., xvi. 21.

The last two lines give the mean results in logarithms and in magnitudes.

TABLE V. — Comparison Stars of β Persei.

Name.	Gr.	No. Nights.	Mag.	P. E.	Log.	S-C.	W-C.	HC-C	HC-C'	HC-Curve.
γ Andromedæ.	23.4	11	1.89	.05	9.085	+.007	—.010	+.054	+.015	.000
β Persei......	20.8	13	2.05	.03	9.021	—	—	+.045	+.017	.000
ι Aurigæ	17.3	8	2.40	.12	8.881	—.181	—.075	+.003	—.011	.000
β Arietis	16.7	12	2.48	.05	8.849	+.033	—.002	—.015	—.027	—.006
ϵ Persei	12.8	10	2.75	.06	8.741	+.034	—.020	—.025	—.021	—.002
γ Persei	10.9	10	2.85	.03	8.701	—.019	—.027	—.017	—.006	—.007
β Trianguli...	9.1	14	2.86	.04	8.697	+.042	+.042	+.023	+.042	+.003
δ Persei	7.8	11	2.90	.05	8.681	+.100	+.053	+.040	+.064	+.011
α Trianguli ...	3.5	12	3.26	.05	8.537	—.003	+.054	+.003	—.016	—.001
ν Persei	0.9	10	3.71	.06	8.357	—	—.038	—.111	—.060	.000
Mean in logarithms				±.020		±.052	±.045	±.034	±.028	±.003
Mean in magnitudes				±.05		±.18	±.11	±.08	±.07	±.01

The eighth and ninth columns show that the agreement of our measures with the estimates of Schönfeld is better than that of either Seidel or Wolff. This is the case even when the value of g is derived from the measures of these observers. The last column shows that a curve could be made to follow the observations almost exactly, the small deviations being allowed rather than that too sharp a change of curvature should be given to the curve.

The form of light-curve deduced from the above measures is shown in Table VI. The first column gives the time, and the second the corresponding magnitude, found by reading the ordinates of the curve drawn through the observed points as described above. A correction to the ephemeris of thirty-seven minutes in the time of minimum is assumed, and the points correspond to intervals of thirty minutes from this time. The logarithm of the light is given in the third column, and is found by multiplying the magnitudes by 0.4 and subtracting 1.068. The relative intensity of the light compared with the full brightness is given in the next column. The observations of Schönfeld are next reduced by assuming the value of g to be successively 0.025 and 0.029; and, thirdly, by means of the curve described on page 380. The residuals in the last three columns are found by subtracting the logarithms given in the third column from these three sets of values.

TABLE VI. — Light Curve of β Persei.

Time.		Mag.	Log.	Light.	Schönfeld.		
					$g = .025$	$g = .029$	Curve.
—4	30	2.67	0.000	1.000	—	—	—
4	0	2.67	0.000	1.000	—.014	—.016	—.023
3	30	2.60	.972	.938	—.002	—.007	—.018
3	0	2.50	.932	.855	+.015	+.007	—.013
2	30	2.35	.872	.745	+.042	+.028	—.012
2	0	2.18	.804	.637	+.058	+.036	—.018
1	30	1.98	.724	.530	+.057	+.022	—.014
1	0	1.80	.652	.449	+.040	—.009	+.008
—0	30	1.68	.604	.402	+.032	—.025	+.010
0	0	1.64	.588	.387	+.031	—.030	+.005
+0	30	1.68	.604	.402	+.031	—.027	+.008
1	0	1.79	.648	.445	+.022	—.031	—.004
1	30	1.94	.708	.510	+.017	—.027	—.025
2	0	2.12	.780	.603	+.029	—.002	—.051
2	30	2.26	.836	.686	+.038	+.019	—.037
3	0	2.39	.888	.773	+.035	+.023	—.012
3	30	2.48	.924	.840	+.036	+.030	+.012
4	0	2.56	.956	.904	+.030	+.028	+.022
4	30	2.63	.984	.964	+.016	+.016	+.016
5	0	2.66	.996	.991	+.004	+.004	+.004
+5	30	2.67	0.000	1.000	—	—	—
Mean in logarithms					±.029	±.019	±.016
Mean in magnitudes....................					±.07	±.05	±.04

It does not seem to be practicable to obtain at present more accurate values from the observations of Dr. Schönfeld, on account of the uncertainty in the value of a grade. The observations themselves are much more precise, and determine the time of minimum, as has been shown above, with an accuracy nearly equal to that of the photometric measures. Even if more accurate measures of the comparison stars should be made, we should still be in doubt whether to assume that g is constant, or that the reduction should be made, as in the last column, by a curve. From the residuals it appears that the various deduced values differ from each other more than they differ from the photometric measures. It accordingly appears scarcely safe to correct the latter by the former. The three values of the minimum corresponding to the last three columns are 1.72, 1.56, and 1.65 magnitudes, their mean agreeing exactly with the photometric measure of 1.64.

It is to be noticed that the value of $g = 0.029$ is confirmed by the photometric measure of β *Persei*, since the residuals are less than when g is taken equal to 0.025. A wholly independent test of the accuracy of the meridian photometer measures is thus afforded. Since

the residuals are smallest in the last column, it seems probable that the value of a grade is not always the same.

The results of the two methods agree as closely as would be expected, even if no systematic errors increased their discordance. The residuals of the photometric observations indicate a probable error of 0.024 magnitudes for each group. Assuming an equal accordance in the observations of Schönfeld, the two methods should differ by 0.04, or by the amount found in Table VI.

DM. 81°25.

The variability of this star was detected by M. Ceraski, of Moscow, during the summer of 1880. It was soon shown that it belonged to the Algol class, or that every few days it lost a large portion of its light for several hours; the interval in the case of this star is somewhat less than two days and a half. Measurements of its light were made according to the method described above in the case of *β Persei*. The photometer was attached to the 15-inch telescope of the Harvard College Observatory, since as much light-gathering power as possible was desired, owing to the faintness of the star. The same observers took part in the work, and the observations were made in the same way as with *β Persei*, except that the images were reversed by turning the photometer instead of by moving the prism. This could be done very conveniently by a pinion, which served to rotate the entire tail-piece of the telescope. The prism was therefore set once for all, and the images reversed and separated by any desired amount with great nicety by turning a milled head. The star DM. 81°26, which is estimated in the *Durchmusterung* to be of the 9.5 magnitude, and is nearly north at a distance of 5′, was used for comparison. DM. 81°30 would have been better on account of its greater brightness, but its distance of 8′ is so great that both images could not be easily brought together. The large angle of the prisms and their distance from the object-glass rendered the light-pencils divergent. At first this gave much trouble, but it was remedied by placing the images always in the same part of the field. Two cardboard points visible against the background of the sky secured this condition. The great northern declination reduced the errors of the driving clock to about one sixth of what they would be for an equatorial star.

The first measures to determine the full brightness of the variable in terms of that of the comparison star were made on February 6, 1881. On the following evening the variable attained its minimum at about half-past eleven. Forty sets or four hundred and eighty settings were obtained between a quarter past six and half past ten. The

later observations were made through clouds which finally stopped the measurements. On February 17 seventy-five sets or nine hundred settings were obtained; the observations extended from seven o'clock in the evening until the variable had regained its full light, at about half-past two on the following morning. During this time no interval of more than five minutes elapsed during which an observer was not comparing the two images. During most of the time the observers took sets alternately, so that there was only an interval of a few seconds between the sets. On February 22 observations began at half past six and continued until ten o'clock, when they were stopped by clouds. Twenty-six sets were obtained in this time. A long period of cloudy weather intervened, and the next measures were made on March 24. Thirty-six sets were taken through clouds, from quarter-past nine to quarter-past twelve. Owing to the small distance between the stars, no perceptible error seems to be introduced by these clouds, as long as they are not dense enough to render the stars invisible. Some measures were obtained on March 14, but apparently the wrong star was observed. They were stopped by the deposition of dew on the object-glass, which may have caused an error, since the two pencils include different portions of the objective. No use has been made of these observations. On April 3 another minimum was observed. Fifty-two sets of six hundred and twenty-four settings were made between seven o'clock and midnight, when the star had recovered its full brightness. Forty-four sets of five hundred and twenty-eight settings were also made on other evenings to determine the undiminished light of the star. Fifteen of these sets by Mr. Searle give its light as 3.64 magnitudes brighter than DM. 81°26. Sixteen sets by Mr. Wendell gave 3.59, and thirteen sets by myself gave 3.71. As the evidence of systematic difference is not conclusive, the mean of all, or 3.64, has been adopted.

The entire number of measures, not including those of March 14, is 273 sets or 3276 settings.

Table VII., like Table I., gives the results of these measures arranged in groups in the order of times from the computed minimum. The columns give the mean of the times, the number of sets of twelve settings, the mean magnitude, the corresponding magnitude derived from a curve drawn nearly through them, and the difference of the last two columns. Each group extends over thirty minutes, except the first, which extends from —311 to —258, and the last three, which include all the measures made when the nearest minimum was more than five hours distant. Their limits are +852 to +991, +1350 to +1495, and +1887 to +2250 minutes.

TABLE VII. — LIGHT-CURVE OF DM. 81°25.

Time.	No.	Obs.	Curve.	O — C.
— 286	6	3.39	3.45	—.06
237	8	3.30	3.27	+.03
208	14	3.07	3.09	—.02
178	11	2.91	2.85	+.06
148	15	2.44	2.44	.00
118	14	1.81	1.80	+.01
89	14	1.34	1.33	+.01
62	14	1.27	1.25	+.02
31	12	1.16	1.24	—.08
— 3	9	1.29	1.24	+.05
+ 31	12	1.28	1.24	+.04
60	14	1.32	1.32	.00
90	18	1.93	1.93	.00
120	17	2.48	2.48	.00
150	14	2.94	2.93	+.01
179	14	3.22	3.23	—.01
207	15	3.42	3.42	.00
237	8	3.54	3.54	.00
922	12	3.66	3.64	+.02
1411	19	3.65	3.64	+.01
+2127	18	3.62	3.64	—.02
	273			±.019

From the last three groups we may infer that the light of this star, like that of the others of the same class, is constant except during the few hours immediately preceding or following the minimum.

The same precautions were taken as with *β Persei* in drawing the light-curve that it should be free from sudden changes in curvature. From the small residuals in the last column we may therefore infer that the accidental errors are very small. About an hour before the minimum the light ceases to vary, and remains nearly constant for an hour and a half, when it begins to rapidly increase. The exact time of these changes may be found more precisely by subdividing the groups whose means are —89 and +60. Making the period of the groups ten minutes instead of thirty, we replace the first group by three containing 4, 5, and 5 sets, having mean times 99, 90, and 80, and magnitudes 1.54, 1.30, and 1.21. The other group similarly subdivided gives for the mean times 52, 60, and 69, the magnitudes, 1.25, 1.26, and 1.44. From these we might infer a somewhat longer period of uniform light than would be indicated by the curve already drawn. The number of observations is, however, too small to determine this point with certainty.

The correction to the ephemeris of the minima is best found by Table VIII., which gives the time at which the light is equal while

decreasing and while increasing. The successive columns give the light in magnitudes, the corresponding times before and after the minimum, and the mean of these times.

TABLE VIII. — TIME OF MINIMUM.

Magn.	Dec.	Inc.	Mean.
3.6	—362	+256	—53.0
3.5	—305	+225	—40.0
3.4	—270	+204	—33.0
3.3	—246	+190	—28.0
3.2	—226	+175	—25.5
3.1	—209	+164	—22.5
3.0	—194	+155	—19.5
2.9	—182	+147	—17.5
2.8	—173	+140	—16.5
2.7	—164	+133	—15.5
2.6	—157	+127	—15.0
2.5	—151	+121	—15.0
2.4	—146	+115	—15.5
2.3	—141	+109	—16.0
2.2	—137	+104	—16.5
2.1	—132	+ 99	—16.5
2.0	—128	+ 94	—17.0
1.9	—123	+ 88	—17.5
1.8	—117	+ 83	—17.0
1.7	—112	+ 78	—17.0
1.6	—107	+ 73	—17.0
1.5	—101	+ 68	—16.5
1.4	— 94	+ 64	—15.0
1.3	— 87	+ 59	—14.0

From the numbers in the last columns we may infer a correction of 13 minutes when the light equals 1.24, or at the minimum. In other words, thirteen minutes should be subtracted from the adopted ephemeris of the minima. The minimum can evidently be determined with much precision from any observations of the times at which the light is equal when diminishing and increasing. If the light is less than 2.9, or the interval between the times less than five hours and a half, it is only necessary to take the mean of the two times and subtract from two to four minutes. The exact correction is found from the last column of the table after subtracting thirteen minutes. The observation is easily made with a small telescope, as there are so many comparison stars of suitable brightness near the variable. Doubtless a very precise determination of the minimum could thus be easily obtained.

To reduce the separate observations we must determine the rate of change in light. The method employed for β *Persei* has again been used; the results are given in Table IX. The columns have the same meaning as in Table III.

TABLE IX. — RATE OF CHANGE IN LIGHT.

Time.	Decreasing.			Increasing.		
	Obs.	Curve.	O — C.	Obs.	Curve.	O — C.
300	—	—.0003	—	—	+.0004	—
275	—	—.0017	—	—	+.0012	—
250	—.0040	—.0032	—.0008	+.0026	+.0024	+.0002
225	—.0057	—.0050	—.0007	+.0040	+.0040	.0000
200	—.0067	—.0073	+.0006	+.0063	+.0054	+.0009
175	—.0106	—.0108	+.0002	+.0086	+.0088	—.0002
150	—.0180	—.0171	—.0009	+.0125	+.0125	.0000
125	—.0198	—.0201	+.0003	+.0166	+.0162	+.0004
100	—.0157	—.0152	+.0005	+.0188	+.0196	—.0008
75	—.0025	—.0024	—.0001	+.0205	+.0198	+.0007
50	.0000	—.0006	+.0006	+.0026	+.0030	—.0004
25	.0000	.0000	.0000	.0000	.0000	.0000
0	.0000	.0000	.0000	.0000	.0000	.0000

The greatest change in light amounts to two hundredths of a magnitude a minute, or at the rate of a magnitude and two tenths an hour. This is much greater than the change of any other known variable, being over three times that of *β Persei*. Accordingly, we should expect a corresponding increase in the accuracy with which the time of minima could be determined.

The observations of DM. 81°25 are grouped in Table X. The successive columns, like those of Table IV., give a current number, the condition limiting the group, the number of sets, the arithmetical sum of the residuals, their algebraic sum giving to each the sign of R divided by a, and the correction to be inferred, or ΔR divided by Σa. The remaining columns give the probable error, the average of the residuals, and the average difference of the three sets of four contained in each set of twelve settings.

TABLE X. — COMPARISON OF RESULTS.

No	Group.	No. Sets.	Σa	ΣR	ΔR	$\frac{\Delta R}{\Sigma a}$	Prob. Err.	Av. Dev.	Av. Resid.
1	Feb. 7....	38	.3463	3.31	— .35	—1.0	1.3	.087	.065
2	" 17....	62	.7071	7.55	+ .01	0.0	1.1	.121	.063
3	" 22....	23	.2408	2.49	— .01	0.0	1.9	.108	.082
4	March 24..	36	.4312	3.03	+ .35	+0.8	1.0	.084	.059
5	April 3....	38	.4000	3.22	+ .74	+1.8	1.1	.084	.051
6	Obs. of P..	76	.8618	6.41	— .01	0.0	0.7	.084	.055
7	" S..	69	.7248	9.11	+1.69	+2.3	1.3	.132	.072
8	" W..	52	.5388	4.08	— .94	—1.7	0.8	.078	.062
9	Decrease ..	91	.9089	9.23	+ .39	+0.4	0.9	.101	.071
10	Increase ..	106	1.2165	10.37	+ .35	+0 3	0.6	.097	.055
11	Total......	197	2.1254	19.60	+ .74	+0.3	0.6	.099	.062

The average probable error of the five minima observed is 1.3 minutes, or about one third of that of *β Persei*. This ratio, as has been already stated, was to be expected, since the rate of variation of the stars is about as three to one. The average deviations from the ephemeris, after applying the constant correction of thirteen minutes, is only 0.7 minutes. It becomes still less if we adopt another ephemeris, as will be shown below. Clouds or twilight prevented observations on both sides of the minimum on every night except on February 17. Accordingly, from a complete observation of a minimum under favorable circumstances we may expect an error of but a few tenths of a minute.

The systematic difference between the observers is found by dividing the algebraic sum of the residuals of each by their number: the algebraic sum of 86 residuals by Mr. Searle is —5.30; of 85 by Mr. Wendell, —0.87; and of 102 by myself, +5.62. The corresponding corrections are, —0.06, —0.01, and +0.06. As each of these represent over a thousand settings, the differences are not probably due to accident. The excess of the computed probable error in the eighth column of Table X. over that to be inferred from the residuals in the seventh column is partly due to the neglect of these differences. If applied to the observations, they would make them appear more accordant. They would not probably sensibly affect the form of light-curve or the times of minima, owing to the distribution of the measures of each observer.

The variation in light is given in Table XI., which is derived from the light-curve described above, after applying a correction of thirteen minutes to the assumed minimum. The ratios of light are given for every half-hour, expressed in differences of magnitude, in logarithms, and in numbers, the full brightness being assumed as the unit in the last two columns.

Some interesting theoretical deductions may be drawn from this light-curve. For about an hour and a half the light remains sensibly constant at 0.110, or about one ninth of its full intensity. This interval is over one third of that during which the light is increasing or diminishing. If the variation in light is admitted to be due to a dark, eclipsing satellite, the diameter of the latter must be $\sqrt{1 - 0.110} = 0.943$ of that of the star, in order to sufficiently reduce the light. A somewhat less diameter is possible if we admit that the star, like our sun, is darker near the edges than in the centre. The effect of this is probably slight, or it would show itself in other ways. The longest period of uniform minimum light would occur if the satellite produced a central

TABLE XI. — Light-Curve of DM. 81°25.

Time.	Mag.	Log.	Light.
h. m.			
−6 30	3.64	0.000	1.000
6 00	3.62	9.992	0.982
5 30	3.58	9.976	0.946
5 00	3.52	9.952	0.895
4 30	3.44	9.920	0.832
4 00	3.34	9.880	0.759
3 30	3.19	9.820	0.661
3 00	2.99	9.740	0.550
2 30	2.68	9.616	0.413
2 00	2.12	9.392	0.247
1 30	1.71	9.228	0.169
1 00	1.27	9.052	0.113
−0 30	1.24	9.040	0.110
0 00	1.24	9.040	0.110
+0 30	1.24	9.040	0.110
1 00	1.26	9.048	0.112
1 30	1.67	9.212	0.163
2 00	2.26	9.448	0.281
2 30	2.76	9.648	0.445
3 00	3.13	9.796	0.625
3 30	3.35	9.884	0.766
4 00	3.51	9.948	0.887
4 30	3.59	9.980	0.955
+5 00	3.64	0.000	1.000

annular eclipse. In this case, if the motion was uniform, the duration of the minimum light would equal only one ninth of that of increase or decrease. The effect of the curvature, or ellipticity, of the path would not greatly affect this conclusion. A very great ellipticity is not admissible, or at the periastron the satellite would strike the star. We are therefore obliged to admit that the eclipse is total (that is, that the star is entirely covered by the satellite), and that the light during the minima is due to one of the two following causes: first, that the satellite is self-luminous, and that the light at the time of the minimum is that received from the satellite, the star itself being completely obscured. In this case we should expect to find a corresponding diminution midway between the minima when the star was in front of the satellite, and accordingly cut off a portion of its light. The loss of light would, however, be small, and might easily escape detection. The greatest effect would occur when the transit was central. In this case, to produce the observed duration of the minimum, assuming the motion to be uniform, the diameter of the satellite should be about 1.3, that of the star being taken as unity. Since the light of the satellite is supposed to be 0.110, that of the satellite and star together being taken as unity, it follows that if the star passes in front of the satel-

lite, it will cut off $\frac{1}{(1.3)^2}$ of its light, or produce a diminution in the total light of $\frac{1}{(1.3)^2} \times 0.110 = 0.065$. The secondary minimum would therefore reduce the light from 1.000 to $1.000 - 0.065 = 0.935$, or about 0.07 magnitudes. This will be the greatest effect, and would be less if the transit was not central. An eccentricity in the orbit of the satellite might even reduce it to zero by carrying the satellite at superior conjunction entirely to one side of the star.

The light reflected by the satellite from the star does not account for this phenomenon, since during its transit the dark side of the satellite would be turned toward the observer. In no case would the light reflected be sufficient, because the satellite does not receive one ninth of the light of the primary; so that, even if all were reflected, it could not emit a sufficient amount of light.

A second hypothesis would explain the prolonged diminution of light by admitting that the satellite consisted of a cloud of meteors so scattered that about 0.110 of the light could pass through the central portions. We should then expect that somewhat more light would pass through the edges, and accordingly that the light would vary slightly during the whole obscuration, attaining a true minimum when the centres of the star and satellite appeared to coincide.

In a recent note Dr. Vogel informs me that he has found no perceptible approach or recession of Algol by means of the spectroscope.* If this observation is confirmed with the other similar variables, we should infer that the masses of the eclipsing satellites were small, or that the second hypothesis is the more probable of the two. In any case, an excellent example is afforded of the value of indirect observations, like those with the photometer or spectroscope, in solving certain problems where direct measurements are valueless.

The next step is to compare the results of other observers, and to derive the correction to the ephemeris. Following the example of Argelander by reducing to Paris mean time, the ephemeris for the time at which any minimum will occur may be expressed by the formula, —

$$\text{Time of minima} = 1880 \text{ June } 28^d\, 7^h\, 44.0^m + 2^d\, 11^h\, 50.0^m\, E.$$

The number of minima which have elapsed since the discovery of the variability is here designated by E, which accordingly equals zero

* See also Ber. der Königl. Säch. Gesell. xxv. 555, and Proc. Amer. Acad. xvi. 34.

on June 23, 1880. The first observations which can be reduced are those made by M. Glasenapp* on July 3, 1880. He adopted a series of comparison stars, which will probably be employed by other observers of this variable. Table XII. gives their Durchmusterung designations, and their right ascension, declination, and magnitudes taken from that catalogue. The next columns give the designation by Glasenapp, and the assumed light in grades. Measures of these stars were made on three evenings at the Harvard College Observatory with Photometer I.† attached to a telescope of four inches aperture. These measures must be regarded as provisional; a much more precise determination of their light will probably be obtained next year with a large meridian photometer. From these measures, which are given in the seventh column, the grades of M. Glasenapp are reduced to magnitudes by the formula, $m = 9.5 - 0.07\,g$, in which g denotes the number of grades and m the corresponding magnitude. The results are given in the eighth column. The last two columns give the residuals found by subtracting the H. C. measures from the magnitudes of M. Glasenapp and of the Durchmusterung.

TABLE XII. — COMPARISON STARS FOR DM. 81°25.

DM.	R.A.		Dec.		Mag.	Desig.	Gr.	*H. C.*	*G.*	*G-HC.*	*DM-HC.*
	m.	s.	°	′							
80°23	41	14	80	57.8	9.2	c	0.0	9.6	9.5	—0.1	—0.4
81°22	42	04	81	07.5	9.2	d	1.6	9.4	9.4	0.0	—0.2
80°22	40	28	80	58.3	9.2	b	2.0	9.2	9.4	+0.2	0.0
80°21	39	05	80	48.9	8.9	a	4.6	8.9	9.2	+0.3	0.0
81°27	50	56	81	19.3	8.6	(3)	12.1	8.5	8.6	+0.1	+0.1
81°29	51	35	81	28.1	8.6	(4)	12.6	8.8	8.6	—0.2	—0.2
81°18	38	28	81	10.5	7.6	(5)	17.6	7.5	[8.3]	[+0.8]	+0.1
81°30	52	29	81	10.9	8.3	(2)	18.3	8.1	8.2	+0.1	+0.2
81°25	49	39	81	05.6	7.5	—	—	6.9	—	—	+0.6

The star DM. 81°18 is either variable, or its light in grades is erroneously given by M. Glasenapp. An examination on different evenings showed that it was decidedly brighter than 81°30. This is confirmed by the measures and by the Durchmusterung magnitudes. If the light in grades was written 17.6 by mistake for 27.6, the magnitude becomes 7.6 instead of 8.3, and the residual + 0.1 instead of + 0.8. This cannot be a typographical error, since the stars were arranged by M. Glasenapp in the order of brightness, and 81°18 is

* Astron. Nach., xcviii. 61.
† Annals, xi. p. 7, figs. 5 and 6.

placed before 81°30. The other residuals show a good agreement between the estimates and measures. The Durchmusterung magnitudes also agree well, if we correct for the difference of scale, which makes the residuals for faint stars negative and for bright stars positive.

The individual comparisons by M. Glasenapp are detailed in Table XIII., which gives a current number, the Moscow mean time, and the corresponding light in grades. By the formula $1.00 + 0.07g$ these are reduced to the same scale of magnitudes as that used in measuring the light in Table XI. The results are given in the fourth column. The next column gives the time of minimum derived from each of these observations by means of the light-curve adopted in Table XI. The last column gives the error in the observation of M. Glasenapp, if we assume the minimum to have occurred at $9^h\ 47^m$ Moscow mean time.

TABLE XIII. — M. GLASENAPP'S COMPARISONS OF DM. 81°25 ON JULY 3.

No.	M.M.T.	Gr.	Log.	Time Min.	O — C.
				h. m.	
1	10 40	4.1	1.28	[9 34]	+.03
2	43	2.2	1.15	—	—.10
3	45	4.8	1.34	[9 31]	+.08
4	50	3.4	1.24	—	—.02
5	55	3.6	1.25	[9 59]	—.03
6	57	4.8	1.34	[9 43]	+.05
7	11 1	5.4	1.38	9 45	+.04
8	8	6.8	1.48	9 47	.00
9	12	8.1	1.57	9 47	.00
10	15	8.8	1.62	9 48	—.01
11	17	9.3	1.65	9 46	—.02
12	19	12.3	1.86	9 40	+.15
13	22	10.4	1.73	9 47	—.05
14	24	13.5	1.94	9 40	+.13
15	27	11.8	1.83	9 49	—.05
16	31	12.2	1.85	9 52	—.10
17	37	13.2	1.92	9 54	—.14
18	41	14.2	1.99	9 55	—.15
19	51	14.5	2.02	[10 04]	[—.30]
20	59	14.5	2.02	[10 12]	[—.44]
21	12 23	16.2	2.13	[10 29]	[—.72]
22	47	17.2	2.20	[10 50]	[—.93]
23	13 23	17.4	2.32	[11 19]	[—1.08]

The error in the estimated light of DM. 81°18 seems to have affected the last measures of the variable. It would appear that after increasing for over two hours (or three hours after the minimum), the variable had not attained the brightness of DM. 81°30, or 18.3 grades! The last five observations have accordingly been bracketed. The

second and fourth comparisons cannot be reduced, since the light is less than that adopted for the minimum. All of those preceding eleven hours have also been bracketed, since the variation in light is so small that an exceedingly small weight should be assigned to them. Retaining them would not sensibly affect the result. The mean of the remaining twelve gives for the time of minimum $9^h\ 47^m$ Moscow mean time. The proximity of the minimum does not affect the residuals of the last column. The last five are alone rejected, since, owing to the cause stated above, they indicate errors too large to be accidental. The mean of the eighteen residuals retained is 0.06 magnitudes.

The most complete series of naked-eye observations of this variable are those of Dr. Schmidt of Athens. Five minima were observed by him in August.* As all the comparisons were made after the period of least light, he was obliged to wait for their reduction until October 8, when he observed the star both before and after the minimum. The first reduction of these observations was made from a curve derived from the measures of October 8. Later he has given a discussion of thirteen minima,† from which he infers a rapid increase in the period. In this paper he omits the observations of August 22, although in his former paper he had assigned to it and to the minimum of August 17 weights double those of any of the others. No reason is given for this omission. There also seems to be a misprint in line 12, p. 89, of this same article. December 7 should apparently be December 2, as this date is employed below. Otherwise, an error of nineteen minutes would be indicated in the observed minimum. A second reduction is given of the August observations, by which the time of minimum is increased more than half an hour. As the original comparisons have not been published, it is impossible to rediscuss them. As the star varies only a few hundredths of a magnitude during nearly two hours, it is obvious that large differences may arise in the assumed time when the light is least. Dr. Schmidt has also determined the period by a method free from this criticism. He has compared the intervals between the times at which the variable equals one of the comparison stars in brightness. Unfortunately, he has not stated the times at which this occurs, so that a comparison with other observers is not practicable.

Mr. George Knott has also observed seven of the minima by the method of Argelander.‡ On September 23 and 28 the variable was

* Astron. Nach., xcviii. 283. † Astron. Nach., xcix. 87.

‡ Astron. Nach., xcix. 109. Nature, xxiii. 542.

observed at the Harvard College Observatory by the same instrument which was used in determining the light of the comparison stars. The image of α *Ursae Minoris* was rendered equal to the variable and to DM. 81°30 alternately. Seven settings were made in each set, beginning and ending with the variable. Systematic errors were thus greatly reduced, since those only would enter which affected one star and not the other. Although a large number of readings were taken, the results were not satisfactory and probably have but little value. The result for September 23 was $8^h\ 57^m$, Cambridge mean time, and for September 28, $8^h\ 7^m$, the difference between the two being five days less fifty minutes instead of five days less twenty minutes. On neither evening were observations obtained before the minimum, owing to twilight.

TABLE XIV.—COMPARISON OF OBSERVED MINIMA.

No.	E.	Comp. Time.				Obs.		Meridian.	O — C.	Authority.
			d.	h.	m.	h.	m.		m.	
1	0	June	23	7	44	—	—	—	—	Ceraski.
2	4	July	3	7	04	9	47	Moscow.	+22	Glasenapp.
3	16	Aug.	2	5	04	7	10	Athens.	+40	Schmidt I.
4	"	"		"		7	53	"	+83	Schmidt II.
5	18	Aug.	7	4	44	6	54	"	+44	Schmidt I.
6	"	"		"		7	30	"	+80	Schmidt II.
7	20	Aug.	12	4	24	6	15	"	+25	Schmidt I.
8	"	"		"		7	03	"	+73	Schmidt II.
9	22	Aug.	17	4	04	5	41	"	+11	Schmidt I.
10	"	"		"		6	28	"	+58	Schmidt II.
11	24	Aug.	22	3	44	5	21	"	+11	Schmidt I.
12	37	Sept.	23	13	34	8	57	Cambridge.	+17	H. C. O.
13	39	"	28	13	14	8	07	"	—13	"
14	43	Oct.	8	12	34	14	16.5	Athens.	+16.9	Schmidt.
15	47	"	18	11	54	13	33.6	"	+14	"
16	49	"	23	11	34	11	27	Greenwich.	+02	Knott.
17	51	"	28	11	14	12	44.0	Athens.	+ 4.4	Schmidt.
18	53	Nov.	2	10	54	12	31.0	"	+11.4	"
19	"	"		"		11	0	Greenwich.	+15	Knott.
20	55	Nov.	7	10	34	11	55.3	Athens.	— 4.3	Schmidt.
21	59	"	17	9	54	11	29.3	"	+ 9.7	"
22	61	Nov.	22	9	34	11	5.7	"	+ 6.1	"
23	"	"		"		9	30±	Greenwich.	+ 5	Knott.
24	65	Dec.	2	8	54	9	0	"	+15	"
25	"	"		"		10	25.8	Athens.	+ 6.2	Schmidt.
26	69	Dec.	12	8	14	9	47.7	"	+ 8.1	"
27	79	Jan.	6	6	34	6	36±	Greenwich.	+11	Knott.
28	92	Feb.	7	16	24	10	18.2	Cambridge.	—12.0	H. C. O.
29	96	"	17	15	44	9	37.2	"	—18.0	"
30	98	"	22	15	24	9	17.2	"	—13.0	"
31	110	March	24	13	24	7	16.4	"	—13.8	"
32	112	"	29	13	04	12	45	Greenwich.	—10	Knott.
33	114	April	3	12	44	12	24	"	—11	"
34	"	"		"		6	35.8	Cambridge.	—14.8	H. C. O.

All of the observations are compared in Table XIV. The columns give a current number, E or the number of minima which have elapsed since the discovery of the variability, the date, hour, and minute according to the approximate ephemeris used, and the observed minimum in mean time of the meridian of the observatory named in the fifth column. The last two columns give the correction to the ephemeris and the name of the observer. Observations made at the Harvard College Observatory are designated by H. C. O. Schmidt I. and Schmidt II. denote the two reductions referred to above.

For comparison with different ephemerides it will be convenient to group the observations of each observer, as has been done in Table XV. The observations of Dr. Schmidt in August according to his first and second reduction have been placed together, and also his later observations. The successive columns give a current number, the authority, the number of minima observed, and the mean value of E. The last four columns give the corrections in minutes to be applied to four ephemerides; that is, they equal the mean of the observed minus the computed value of each group according to the following four formulas: —

(A) Ep. E. = 1880 June 23^d 7^h $44.0^m + 2^d$ 11^h 50.0^m E.
(B) Ep. E. = 1880 June 23^d 10^h $13.1^m + 2^d$ 11^h 44.94^m E. + 0.04376^m E^2.
(C) Ep. E. = 1880 June 23^d 8^h $12.0^m + 2^d$ 11^h 49.6^m E.
(D) Ep. E. = 1880 June 23^d 7^h $41.0^m + 2^d$ 11^h 49.9^m E.

The first of these formulas (A) is extremely convenient, since the minutes repeat themselves every six minima. As the period differs from two days and a half by exactly ten minutes, the times of the successive minima may be written down directly. Every two hundred and forty minima, or every five hundred and ninety-nine days, the hours and minutes repeat themselves, so that the ephemeris can be easily extended over long periods. Whatever ephemeris is adopted, it may be more convenient to compute the minima by this formula first and apply the difference of the ephemerides as a correction.

Formula (B) is derived from the law proposed by Dr. Schmidt on page 90 of his article, reducing to Paris mean time and adopting June 23 as the starting-point, as in the other ephemerides. A minimum is assumed to have occurred on December 7 at 10^h 6.7^m, Athens mean time. The period at this time is taken as 2^d 11^h 50.812^m, with an increase of 0.08753 in each successive period. The ephemeris on page 91 is nearly, but not exactly, represented by this law. A part

of the discrepancy is due to an error by which the interval between the minima of October 3 and 6 is about five minutes too small. This will affect all the minima preceding or all of those following it. The number of decimal places employed by Dr. Schmidt has been retained, although the accuracy of the observations does not seem to justify it. Since we do not know the period within some tenths of a minute, it seems scarcely advisable to carry the result to thousandths of a minute in the formula used to represent it.

Formula (C) is proposed as that which best satisfies all the observations, if we admit that the period is invariable.

Formula (D) is that which best represents the later measures obtained at the Harvard College Observatory.

TABLE XV. — COMPARISON OF EPHEMERIDES.

No.	Authority.	No. Min.	E.	A.	B.	C.	D.
1	Glasenapp.	1	4	+22.0	—105.1	— 4 4	+25.4
2	Schmidt I.	5	20.0	+26.2	— 36.3	+ 6.2	+31.2
3	Schmidt II.	4	19.0	[+73.5]	+ 7.8	[+53.1]	+78.4
4	H. C. O.	2	38.0	+ 2.0	— 14.6	—10.4	+ 8.8
5	Schmidt.	9	55.9	+ 8.1	+ 1.3	+ 2.5	+16.7
6	Knott.	5	61.4	+ 9.6	+ 1.5	+ 6.2	+18.7
7	H. C. O.	1	92	—12.0	— 65.7	— 3.2	+ 0.2
8	"	1	96	—13.0	— 79.0	— 2.6	— 0 4
9	"	1	98	—13.0	— 85.7	— 1.8	— 0.2
10	"	1	110	—13.8	—133.9	+ 2.2	+ 0.2
11	Knott.	1	112	—10	—137.2	+ 6.8	+ 4.2
12	"	1	114	—11	—149.7	+ 6.2	+ 3.4
13	H. C. O.	1	114	—14.8	—153.7	+ 2.8	+ 0.4

An examination of the residuals from Dr. Schmidt's formula shows that this ephemeris alone satisfies all of his measures, if we admit his second reduction of his observations in August. It, however, entirely fails to represent the later determinations. The deviations exceed two hours, both in the Harvard College measures and in those of Mr. Knott When this ephemeris was published, these observations had not been made, and of course such a deviation could not have been foreseen. It is, however, remarkable that Dr. Schmidt should not have noticed the large discordance in the minimum observed by M. Glasenapp. This observation has especial value as a test of any ephemeris, since it is much earlier than any other measures. Dr. Schmidt's ephemeris would give a minimum at $11^h\ 32^m$, Moscow mean time, which is at once seen to be in error on inspecting Table XIII. or the original publication of M. Glasenapp. In fact, the reduction shows a correction to the time of minimum of over an hour and a half.

All the observations are fairly represented by formula (C), except the second reduction of those of Dr. Schmidt. His original reduction leaves a residual which might well be due to errors of observation. Although the residuals of the Harvard College measures are small, they are still much larger than their probable errors, and their values evidently indicate systematic error. The last formula satisfies these completely, giving average residuals of only 0.3 minutes, but does not agree with the other observations. If we admit a variation in the period, the value $2^d\ 11^h\ 49.9^m$ would seem to be that between $E = 90$ and $E = 114$, but not between $E = 0$ and $E = 90$.

It is scarcely worth while at present to discuss the relative probability of these various formulas, since further observations which will doubtless soon be made will serve to decide between them with certainty. If the original observations were published, so that all could be reduced according to the same method, doubtless much greater accordance would be found in the results. There seems to be no reason why the error in determining each minimum from observation during the decrease and increase should not be reduced to two or three minutes or much less than those of *β Persei*. Should the discrepancy of certain measures, as those in August of Dr. Schmidt, be confirmed, they would indicate the existence of some disturbing body which might also account for such a deviation as that noted in the minimum of *β Persei* on Nov. 22. No correction has been applied for the aberration. The star is so near the pole of the ecliptic that the correction would never exceed two minutes, and would be masked by the other errors.

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